Dynamics of Fuzzy Argumentation Frameworks

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Abstract. Dung's theory of abstract argumentation plays an incremental role in artificial intelligence. The research about the dynamics theory of argumentation efficiently identifies the justified arguments when arguments or attacks change. However, the dynamics theory is absent in fuzzy argumentation framework (FAF). We want to calculate the semantics of the updated FAF by partially reusing the semantics of the previous FAF. In this paper, we explore the dynamics theory in FAFs. First, we introduce all the changes of FAF, including not only the changes of arguments and attacks but also the increases or decreases of their fuzzy degrees. Thus, the changes in FAFs are more complicated than standard AF. Then by extending Liao's division-based approach, we provide an efficient algorithm for computing some basic semantics. This algorithm conserves part of the semantics in the previous FAF. Thus, we can efficiently compute the belief degree to which arguments are justified.

Keywords: Dynamics of argumentation · Division-based approach · Fuzzy argumentation frameworks · Argumentation semantics

1 Introduction

Dung's theory [7] of argumentation frameworks (AFs) plays an increasingly important role in artificial intelligence and nonmonotonic reasoning. A Dung's AF is essentially a directed graph. The nodes represent the arguments and the arrows represent the attack relation between the arguments. Dung's theory is to seek reasonable subsets of the arguments under some criterions.

In order to handle the uncertain, incompleteness, and inconsistency of information, standard AFs are extended by quantifying arguments or attacks. More specifically, in these quantitative AFs, numerical values are combined with arguments/attacks, such as probabilistic AFs [9,12], fuzzy AFs (FAFs) [5,10,15], weighted AFs [8] and so on. FAFs characterize AFs by fuzzy arguments or fuzzy attack relation. In [10,15], the main task of FAFs is to find the subsets over

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justified fuzzy arguments. The extension semantics to FAFs have been proposed in [10,15]. Changes in arguments and attack relationships are intrinsic to various argumentation systems [2,4,11]. According to some research, arguments and their attack relation develop with the changes in basic knowledge or information or observations [3,6,14]. In [4], Cayrol et al. address the problem of the change of adding an argument in Dung's AF. They focus on the *change* of the argumentation systems and its extension. In [13], Liao et al. proposed a *division-based approach* for dynamics of AFs. The division-based approach provides an efficient algorithm for the dynamics of argumentation systems.

In quantitative AFs, the arguments and attack relation are also changed with the changes of basic knowledge, information, and observations. However, the research about the dynamics of quantitative AFs is absent. In this paper, we take the Gödel FAFs (GFAFs) as an example to explore the dynamics of quantitative AFs. However, the changes in the dynamics of FAFs are more complicated than standard AF. This is because FAF is changed not only by adding (or removing) arguments or attack relation but also by increasing (or decreasing) the belief degree of arguments or attack relation.

The main task of this paper is to provide an efficient algorithm for basic semantics in the updated FAF. We first establish the directionality principle in FAFs. Then we extend the division-based approach, each updated FAF is divided into three parts: unaffected FAF, affected FAF, conditioned FAF. We then compute the extension semantics of the updated FAF by computing the semantics of unaffected FAF and affected FAF under the conditioned FAF. In this way, we can calculate the complete, preferred and grounded semantics which partially reuses the extensions computed in the previous FAF.

This paper is structured as follows: In Sect. 2, we specify the motivation of the dynamics of FAF. In Sect. 3, we review some basic definitions of FAF and fuzzy set theory. In Sect. 4, we explore the various changes in the dynamics of FAFs. In Sect. 5, we extend the division-based method into the dynamics of FAFs. The paper ends with conclusions and remarks about future work.

2 Motivation

As we showed in the Introduction, changes in arguments and attack relationships are intrinsic to various argumentation systems. And compared with standard AFs, the dynamics of FAFs are more complicated. We first specify the intuition of the dynamics of FAFs. To understand the dynamics of FAFs, we consider the following example:

A patient goes to the hospital because of chest tightness. If we only make an empirical judgment about the patient, there are two diseases that may cause chest tightness: coronary heart disease and bronchitis. So we obtain two arguments:

A: The patient's chest tightness is caused by coronary heart disease;
B: The patient's chest tightness is caused by bronchitis.

Assuming that the patient's chest tightness is not caused by these two diseases at the same time. Thus these two arguments are contradictory. We can establish an FAF and the initial belief degree of that the patient's chest tightness is caused by coronary heart disease is 0.4 and the initial belief degree of that the patient's chest tightness is caused by bronchitis is 0.6. If we do a preliminary examination of the patient, the result of the examination shows that the patient has bronchitis and has no history of coronary heart disease. Consequently, the degree of A may naturally decrease and the degree of B may naturally increase, one may change the system by decreasing the degree of A into 0.1 and increasing the degree of B into 0.9. Therefore, in FAF, the change of initial degree of arguments or attack relation also changes the system. In addition, if we take a further examination of this patient, we have that patient suffers from cardiac failure. We then change the systems by adding the fuzzy argument 'the patient's chest tightness may be caused by cardiac failure'.

The dynamics of FAFs in this paper are shown as following:

1. adding arguments that interact with the previous FAF.
2. deleting arguments from the previous FAF.
3. adding attack relation which does not appear in the previous FAF.
4. deleting attack relation from the previous FAF.
5. increasing the initial belief degree of arguments.
6. decreasing the initial belief degree of arguments.
7. increasing the initial belief degree of attack relation.
8. decreasing the initial belief degree of attack relation.

Next, when a fuzzy argumentation system is changed by these above cases, we obtain an updated FAFs. Then, the main task is to find the belief degree to which arguments are justified. Thus, to cope with the semantics of updated FAFs, we extend Liao's division-based approach. By extending Liao's theory, we can compute the complete, preferred and grounded semantics which partially reuses the extensions computed in the previous FAF.

3 Preliminaries

Our work is based on Gödel fuzzy argumentation frameworks [15]. Let's first review the notions of fuzzy set and GFAFs.

3.1 Fuzzy Set Theory

We only show some notions of fuzzy set theory [16] that appear in this paper.

Let X be a nonempty set. A fuzzy set (X, S) is determined by its membership function S: $X \to [0, 1]$, such that for each $x \in X$ the value $S(x)$ is interpreted as the grade of membership of x within X. Given some constant set X, we may denote a fuzzy set (X, S) as S for convenience. A crisp set S' is a classical set, namely for any $x \in X$, $S'(x) = 0$ or $S'(x) = 1$.

A fuzzy set S is contained in another fuzzy set $S^{'}$, if $\forall x \in X$, $S(x) \leq S^{'}(x)$, which is denoted by $S \subseteq S^{'}$.

A fuzzy set S is called a fuzzy point if its support is a single point $x \in X$, and is denoted by $(x, S(x))$. We denote all the support of S as $Supp(S)$, where $Supp(S) = \{x \mid S(x) \neq 0\}$.

A fuzzy point $(x, S(x))$ is contained in a fuzzy set S if it is a subset of S.

The t-norm is a binary operator on $[0, 1]$. In this paper, we focus on Gödel t-norm. Gödel t-norm $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ such that $\forall x, y \in [0, 1]$, $T(x, y) = min\{x, y\}$. For simplify, in this paper, we denote $*$ as Gödel t-norm, namely for any $x, y \in [0, 1]$, $x * y = T(x, y) = min\{x, y\}$.

3.2 Gödel Fuzzy Argumentation Frameworks

In this paper, an FAF consists of fuzzy arguments and fuzzy attack relation between the arguments, and Gödel FAF specializes it using Gödel t-norm.

Definition 1. *A fuzzy argumentation framework is a tuple $\langle \mathcal{A}, \rho \rangle$ where $\mathcal{A} : Args \rightarrow (0, 1]$ and $\rho : Args \times Args \rightarrow (0, 1]$ are total functions. We refer to $\mathcal{A}$ as a fuzzy set of arguments, and ρ as a fuzzy set of attacks, while Args is a crisp set of arguments.*

From [15], we call the elements in $\mathcal{A}$ as fuzzy arguments and the elements in ρ as fuzzy attack. We refer to an FAF using the Gödel t-norm as a GFAF. It is notable that Gödel t-norm is just a composition operator to combine fuzzy arguments and fuzzy attack relation. The FAFs explored in this paper all are GFAFs, and for simplify, we briefly denote GFAF as FAF.

An important distinction between the FAFs and AFs is that the attack relation may have no influence on the choice of acceptable arguments in FAF. We borrow the notions of sufficient attack and tolerable attack from [15].

Definition 2. *Given two arguments (A, a) and (B, b) as well as a fuzzy attack relation $((A, B), \rho_{AB})$, if $a * \rho_{AB} + b \leq 1$, then the attack is tolerable, otherwise it is sufficient.*

A sufficient attack *weakens* the attacked argument. If (A, a) sufficiently attacks (B, b), then (B, b) is weakened to (B, b') by (A, a), where $b' = 1 - a * \rho_{AB}$. We provide the definition of *weakening defend.*

Definition 3. *Given an $FAF = \langle \mathcal{A}, \rho \rangle$, a fuzzy set $S \subseteq \mathcal{A}$ weakening defends a fuzzy argument (C, c) in $\mathcal{A}$ if for any $(B, b) \in \mathcal{A}$ there is some $(A, a) \in S$ such that (A, a) weakens (B, b) to $(B, b^{'})$ and $(B, b^{'})$ tolerably attacks (C, c).*

We provide an alternate definition of *weakening defend.*

Definition 4. *Given an $FAF = (\mathcal{A}, \rho)$, a fuzzy set $S \in A$ weakening defends a fuzzy argument $(A, a) \in \mathcal{A}$ if for any (B, b) sufficiently attacks (A, a) there exists $(C, c) \in S$ such that $c * \rho_{CB} = a$. Namely there exists (C, c) weakens (B, b) to $(B, 1 - a)$ and the attack relation from $(B, 1 - a)$ to (A, a) is clearly tolerable.*

It is notable that if the attack relation from A to B is always tolerable, namely $\mathcal{A}(A) * \rho_{AB} + \mathcal{A}(B) \leq 1$, then the attack relation has no influence in this system. Thus, for simplify, *we don't show this attack relation in this paper.*

We list the extensions semantics in GFAFs as follows.

Definition 5. *Given a $GFAF = \langle \mathcal{A}, \rho \rangle$ and $S \subseteq \mathcal{A}$.*
S is a conflict-free set if all attacks between the arguments in S are tolerable.
A conflict-free set S is an admissible extension if S weakening defends each element in S.
A conflict-free set S is a complete extension if it contains all the fuzzy arguments in $\mathcal{A}$ that S weakening defends.
An admissible extension is a preferred extension if it is maximal.
A complete extension is a grounded extension if it is minimal.
A conflict-free set is stable if it sufficiently attacks every element in $\mathcal{A}$ not in E.

In GFAFs, the grounded extension is unique and it is the least complete extension. The stable extensions coincide with the preferred extensions.

4 Dynamics of Fuzzy Argumentation Frameworks

In this section, we give the definition of *change* in FAFs. The notion of *change* is cited from [4], we introduce all the changes in FAFs. In [4,13], $\mathcal{I}$ denoted the *interactions* between arguments under the context of change. $\mathcal{I}$ represents the changed attack relation. For simplicity, we provide the notion of $\mathcal{I}$ in FAFs.

- $\mathcal{I}_{Ar_1:Ar_2}$ is the set of interactions related to Ar_2 and of the form $((A, B), \rho_{AB})$, $((B, A), \rho_{BA})$, or $((B, B'), \rho_{BB'})$, in which $A \in Ar_1$ and $B, B' \in Ar_2$.
- $\mathcal{I}_{Ar}$ is a set of interactions between the arguments in Ar, and of the form $((A, A'), \rho_{AA'})$, in which $A, A' \in Ar$.
- $\mathcal{I}_{(Ar_1,Ar_2)}$ is the set of interactions from the arguments in Ar_1 to the arguments in Ar_2, and of the form $((A, B), \rho_{AB})$, in which $A \in Ar_1$ and $B \in Ar_2$.

Analogously, we define a form of a set of fuzzy attack relation within FAF:

- ρ_{Ar} is a set of attack relation between the arguments in Ar, and of the form $((A, A'), \rho_{AA'})$, in which $A, A' \in Ar$.
- $\rho_{(Ar_1,Ar_2)}$ is the set of attack relation from the arguments in Ar_1 to the arguments in Ar_2, and of the form $((A, B), \rho_{AB})$, in which $A \in Ar_1$ and $B \in Ar_2$.

Definition 6. *Given an $FAF = \langle \mathcal{A}, \rho \rangle$ and $Supp(\mathcal{A}) = Ar_1$.*

1. *adding a set of fuzzy attack relation $\mathcal{I}_{Ar_1}$ (for any $(A, B) \in Supp(\mathcal{I}_{Ar_1})$, $\mathcal{I}_{Ar_1}(A, B) > \rho(A, B) = 0$) is a change which is defined by:*

$$\langle \mathcal{A}, \rho \rangle \oplus \mathcal{I}_{Ar_1} = \langle \mathcal{A}, \rho \cup \mathcal{I}_{Ar_1} \rangle.$$

2. *removing a set of fuzzy attack relation* $\mathcal{I}_{Ar_1} \subseteq \rho$ *(for any* $(A,B) \in Supp(\mathcal{I}_{Ar_1})$*,* $\mathcal{I}_{Ar_1}(A,B) = \rho(A,B)$*) from FAF is a change which is defined by:*

$$\langle \mathcal{A}, \rho \rangle \ominus \mathcal{I}_{Ar_1} = \langle \mathcal{A}, \rho - \mathcal{I}_{Ar_1} \rangle.$$

3. *adding a set of fuzzy arguments* $\mathcal{B}$ *(*$Supp(\mathcal{B}) = Ar_2$ *and* $Ar_1 \cap Ar_2 = \emptyset$*) which interacts with FAF is a change which is defined by:*

$$\langle \mathcal{A}, \rho \rangle \oplus \langle \mathcal{B}, \mathcal{I}_{Ar_1:Ar_2} \rangle = \langle \mathcal{A} \cup \mathcal{B}, \rho \cup \mathcal{I}_{Ar_1:Ar_2} \rangle.$$

4. *removing arguments* $\mathcal{B} \subseteq \mathcal{A}$ *(*$Supp(\mathcal{B}) = Ar_2$ *and* $\forall A \in Ar_2$*,* $\mathcal{B}(A) = \mathcal{A}(A)$*) from FAF is a change which is defined by:*

$$\langle \mathcal{A}, \rho \rangle \ominus \langle \mathcal{B}, \mathcal{I}_{Ar_1:Ar_2} \rangle = \langle \mathcal{A} - \mathcal{B}, \rho - \mathcal{I}_{Ar_1:Ar_2} \rangle.$$

5. *increasing the initial belief degree of arguments, for simplify, we only increase the initial degree of an argument. We increase the initial degree of A into a, namely we use* (A, a) *replaces* $(A, \mathcal{A}(A))$ *and* $a > \mathcal{A}(A)$*, it is a change which is defined by:*

$$\langle \mathcal{A}, \rho \rangle \oplus (A, a) = \langle \mathcal{A} \cup (A, a), \rho \rangle.$$

6. *decreasing the initial belief degree of arguments, for simplify, we only decrease the initial degree of an argument, we first decrease the initial degree of A into 0, and the we increase the degree of A into a, namely, we use* (A, a) *replaces* $(A, \mathcal{A}(A))$ *and* $a < \mathcal{A}(A)$*, it is a change which is defined by:*

$$\langle \mathcal{A}, \rho \rangle \ominus (A, a) = \langle (\mathcal{A} - (A, \mathcal{A}(A))) \cup (A, a), \rho \rangle.$$

7. *increasing the initial belief degree of attack relation, for simplify, we increase the initial degree of an attack relation. We increase the initial degree of* (A, B) *into* $\rho'(A,B)$*, namely we use* $((A,B), \rho'(A,B))$ *replaces* $((A,B), \rho(A,B))$ *and* $\rho'(A,B) > \rho(A,B)$*, it is a change which is defined by:*

$$\langle \mathcal{A}, \rho \rangle \oplus ((A,B), \rho'(A,B)) = \langle \mathcal{A}, \rho \cup ((A,B), \rho'(A,B)) \rangle.$$

8. *decreasing the initial belief degree of attack relation, for simplify, we only decrease the initial degree of an argument, we first decrease the initial degree of* (A, B) *into 0, and then we increase the degree of* (A, B) *into* $\rho'(A,B)$*, namely, we use* $((A,B), \rho'(A,B))$ *replaces* $((A,B), \rho(A,B))$ *and* $\rho'(A,B) < \rho(A,B)$*, it is a change which is defined by:*

$$\langle \mathcal{A}, \rho \rangle \ominus ((A,B), \rho'(A,B)) = \langle \mathcal{A}, (\rho - ((A,B), \rho(A,B))) \cup ((A,B), \rho'(A,B)) \rangle.$$

Although we only increase the initial degree of an argument in (5)–(8), the case of multiple arguments can be done by iteratively applying the formalism of (5)–(8).

Next, we will define the dynamics of FAFs. Obviously, all the *changes* in Definition 6 are dynamics of FAFs. Additionally, the arbitrary combinations of 1–8 are also the dynamics of FAF. We introduce the dynamics of FAF when combined with an addition of FAF.

Definition 7. *Let* $FAF = \langle \mathcal{A}, \rho \rangle$, *where* $\mathcal{A} : Ar_1 \rightarrow (0,1]$ *and* $\rho : Ar_1 \times Ar_1 \rightarrow (0,1]$ *are total functions. An addition of FAF is represented as a tuple* $(\mathcal{B}, \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2})$, *in which* $\mathcal{B}$ *is a set of fuzzy arguments to be added and* $Ar_2 = Supp(\mathcal{B})$, $\mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2}$ *is a set of fuzzy attacks to be added.*

In the above definition, we have some explanations about the addition of FAF. As far as the addition of fuzzy arguments $\mathcal{B}$ is considered, for each fuzzy argument $(A, a) \in \mathcal{B}$, there are two cases:

- $A \notin Ar_1$, then it coincides with the case 3 in Definition 6;
- $A \in Ar_1$, but $\mathcal{B}(A) > \mathcal{A}(A)$, then it coincides with the case 5 in Definition 6.

For each attack relation $(A, B) \in Supp(\mathcal{I}_{Ar_1}) \cup Supp(\mathcal{I}_{Ar_1:Ar_2})$, there are also two cases:

- $(A, B) \in Supp(\mathcal{I}_{Ar_1:Ar_2}) \backslash Supp(\mathcal{I}_{Ar_1})$, then it coincides with the case 1 in Definition 6;
- $(A, B) \in Supp(\mathcal{I}_{Ar_1})$, but $\rho(A, B) < \mathcal{I}_{Ar_1}(A, B)$, then it coincides with the case 7 in Definition 6.

From the above definition, an updated FAF with respect to an addition FAF is defined as follows:

Definition 8. *Let* $FAF = \langle \mathcal{A}, \rho \rangle$, *where* $\mathcal{A} : Ar_1 \rightarrow (0,1]$ *and* $\rho : Ar_1 \times Ar_1 \rightarrow (0,1]$ *are total functions. Let* $(\mathcal{B}, \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2})$ *be an addition. The updated FAF w.r.t.* $(\mathcal{B}, \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2})$ *is represented as follows:*

$$\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle = \langle \mathcal{A}, \rho \rangle \oplus (\mathcal{B}, \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2}) =_{def} \langle \mathcal{A} \cup \mathcal{B}, \rho \cup \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2} \rangle$$

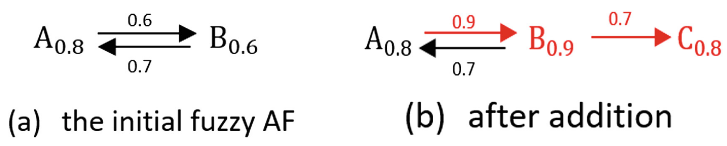

Fig. 1. An example of updated FAF w.r.t. an addition of FAF (Example 1)

We provide an example to illustrate the above definition.

Example 1. *Let* $FAF = \langle \{(A, 0.8), (B, 0.6)\}, \{((A, B), 0.6), ((B, A), 0.7)\} \rangle$. *Suppose* $(\mathcal{B}, \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2})$ *be an addition, in which* $\mathcal{B} = \{(B, 0.9), (C, 0.8)\}$ *and* $\mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2} = \{(A, B), 0.9), ((B, C), 0.7)\}$. *In Fig. 1, the arrows and nodes in red represent changed arguments and attack relation. Then we obtain an updated FAF* $\langle \mathcal{A} \cup \mathcal{B}, \rho \cup \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2} \rangle = \langle \{(A, 0.8), (B, 0.9), (C, 0.8)\}, \{((A, B), 0.9), ((B, A), 0.7), ((B, C), 0.7)\} \rangle$.

5 The Argumentation Semantics of Dynamic Fuzzy Argumentation Frameworks

In this section, we will extend the division-based approach in [13] to the dynamics of FAFs. In this paper, we only consider the efficient algorithms for calculating the complete, grounded and preferred semantics.

5.1 The Directionality Principle in FAFs

The division-based approach in [13] is based on the directionality principle, and thus we first extend the directionality principle into FAFs.

The notion of directionality principle is first provided in [1]. Intuitively, under Dung's AF, the justification status of an argument A is only depended on the status of the defeaters of the argument A (which in turn are affected by their defeaters and so on), while the arguments which only receive an attack from A (and in turn those which are attacked by them and so on) should not have any effect on the status of A. Then Baroni et al. extended the directionality principle by considering the unattacked set which doesn't receive attacks from outside. Here, we extend the directionality principle to FAFs. In FAFs, the belief degree of each argument is only depended on the belief degrees of attackers (which in turn are affected by their defeaters and so on).

Definition 9. *Given an $FAF = (\mathcal{A}, \rho)$, a fuzzy set $\mathcal{U} \in \mathcal{A}$ is unattacked if and only if there exists no $A \notin Supp(\mathcal{U})$, $B \in Supp(\mathcal{U})$ such that $(A, B) \in Supp(\rho)$. The set of unattacked sets of FAF is denoted as $\mathcal{US}(FAF)$.*

We also provide the notion of restricted FAF. Let $FAF = \langle \mathcal{A}, \rho \rangle$. The restriction of FAF to $S \subseteq \mathcal{A}$ is $FAF \downarrow_S = \langle S, \rho_S \rangle$. The directionality criterion can then be defined, the semantics extensions of an unattacked set are not affected by the remaining parts of the FAF.

Definition 10. *A semantics S satisfies the directionality principle if and only if for any FAF, $\forall \mathcal{U} \in \mathcal{US}(FAF)$:*

$$\mathcal{AE}_{\mathcal{S}}(FAF, \mathcal{U}) = \mathcal{E}_{\mathcal{S}}(FAF \downarrow_{\mathcal{U}}) \text{ where } \mathcal{AE}_{\mathcal{S}}(FAF, \mathcal{U}) = \{E \cap \mathcal{U} \mid E \in \mathcal{E}_{\mathcal{S}}(FAF)\}$$

Similar to Dung's AF, the complete, grounded and preferred semantics satisfy the directionality principle in FAF. This is because $\forall E \in \mathcal{AE}_{\mathcal{CO}}(FAF, U)$, there exists no fuzzy argument in E is sufficiently attacked by the fuzzy argument outside the unattacked set.

5.2 The Basic Theory of the Division-Based Approach in FAF

According to the definition of directionality principle, under a certain argumentation semantics $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{GR}\}$, the justified belief degree of argument is only affected by its attacker. Thus, as for a certain semantics that is based on the directionality principle, if an argument is not affected by the newly added argument and attack relation, then its justified degree will not change. Therefore, analogous to Liao's division-based theory in standard AF, in the updated FAF, we should identify the unaffected part and the affected part w.r.t. the changed arguments and attack relation. As far as the unaffected part of the updated FAF is concerned, its semantics can be conserved to calculate the semantics of the updated FAF. Thus, the complexity of computing the semantics of the dynamics of FAF might be decreased. Next, we should consider how to calculate the semantics of the affected part. We will Liao's approach by extending the notion of the conditioned part of the updated FAF to handle the problem. Finally, we combined these two parts of semantics and prove the soundness and completeness of the combined semantics. To cope with these problems, we extend the Liao's theory to FAF in the following section.

5.3 Conditioned Fuzzy Argumentation Frameworks

In order to handle the semantics of the updated FAFs, we extend the division-based approach to FAFs. Firstly, we restate the definition of conditioned FAF.

Definition 11. *Given a fuzzy argumentation framework* $FAF_1 = \langle \mathcal{A}_1, \rho_1 \rangle$*, a conditioned fuzzy argumentation framework w.r.t.* FAF_1 *is a tuple*

$$\text{CFAF} = (\langle \mathcal{A}_2, \rho_2 \rangle, (C(\mathcal{A}_1), \rho_{(C(Ar_1), Ar_2)}))$$

in which

- $Ar_1 = Supp(\mathcal{A}_1)$*,* $Ar_2 = Supp(\mathcal{A}_2)$ *and* $C(Ar_1) = Supp(C(\mathcal{A}_1))$*;*
- $\langle \mathcal{A}_2, \rho_2 \rangle$ *is an FAF that is conditioned by* $C(\mathcal{A}_1)$*, in which* $\mathcal{A}_1 \cap \mathcal{A}_2 = \emptyset$*;*
- $C(\mathcal{A}_1) \subseteq \mathcal{A}_1$ *is a nonempty set of fuzzy arguments (called conditioning arguments) that attacks the fuzzy arguments in* $\mathcal{A}_2$*, i.e.,* $\forall A \in C(Ar_1)$*,* $\exists B \in Ar_2$*, s.t.* $(A, B) \in Supp(\rho_{(C(Ar_1), Ar_2)})$*.*

Since $\langle \mathcal{A}_1, \rho_1 \rangle$ is an FAF that independent of $\langle \mathcal{A}_2, \rho_2 \rangle$, we can obtain the reasonable set of FAF_1, i.e., the semantics extensions of FAF_1 is directly obtained by the corresponding criterion. Given a specific extension $E \in \mathcal{E}_{\mathcal{S}}(FAF_1)$, $C(\mathcal{A}_1)[E]$ is also called a condition of $\langle \mathcal{A}_2, \rho_2 \rangle$ under the reasonable extension E of FAF_1. $CFAF[E] = (\langle \mathcal{A}_2, \rho_2 \rangle, (C(\mathcal{A}_1)[E], \rho_{(C(Ar_1), Ar_2)}))$ is called an assigned CFAF. The semantics of an assigned *CFAF* are related to the semantics of conditioning arguments, which are defined as follows:

Definition 12. *Let* $CFAF[E_1] = (\langle \mathcal{A}_2, \rho_2 \rangle, (C(\mathcal{A}_1)[E_1], \rho_{(C(Ar_1), Ar_2)}))$ *be an assigned CFAF w.r.t.* $FAF_1 = \langle \mathcal{A}_1, \rho_1 \rangle$*, in which* $E_1 \in \mathcal{E}_{\mathcal{S}}(FAF_1)$*,* $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{GR}\}$*.*

- *A set* $E \in \mathcal{A}_2$ *of fuzzy arguments is conflict-free if and only if there exists no* (A, a)*,* $(B, b) \in E$ *s.t.* (A, a) *sufficiently attacks* (B, b) *w.r.t.* ρ_2*.*

- *A fuzzy argument* $(A, a) \in \mathcal{A}_2$ *is weakening defended by a set* $E \in \mathcal{A}_2$ *of fuzzy arguments under the condition* $C(\mathcal{A}_1)[E_1]$ *if and only if the following two conditions hold:*
  - $\forall(B, b) \in \mathcal{A}_2$*, if* (B, b) *sufficiently attacks* (A, a)*, then* $\exists(C, c) \in E$ *s.t.* $c * \rho_{CB} = a$*, or* $\exists(D, d) \in C(\mathcal{A}_1)$*, s.t.* (D, d) *is weakening defended by* E_1 *and* $d * \rho_{DB} = a$*;*
  - $\forall(B, b) \in C(\mathcal{A}_1)$*, if* (B, b) *sufficiently attacks* (A, a)*, then* $\exists(C, c) \in E_1$ *s.t.* $c * \rho_{CB} = a$*.*
- *A conflict-free set* E *is admissible if and only if each argument in* E *is weakening defended by* E *under the condition* $C(\mathcal{A}_1)[E_1]$*.*

Definition 13. *Let* $\mathrm{CFAF}[E_1] = (\langle \mathcal{A}_2, \rho_2 \rangle, (C(\mathcal{A}_1)[E_1], \rho_{(C(Ar_1), Ar_2)}))$ *be an assigned* CFAF *w.r.t.* $FAF_1 = \langle \mathcal{A}_1, \rho_1 \rangle$*, in which* $E_1 \in \mathcal{E}_\mathcal{S}(FAF_1)$*,* $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{GR}\}$*. Let* $E \subseteq \mathcal{A}_2$ *be an admissible set of fuzzy arguments.*

- E *is a preferred extension if and only if* E *is a maximal (w.r.t. set-inclusion) admissible set of fuzzy arguments.*
- E *is a complete extension if and only if each argument that is weakening defended by* E *under the condition* $C(\mathcal{A}_1)[E_1]$ *is in* E*.*
- E *is a grounded extension if and only if* E *is the minimal (w.r.t. set-inclusion) complete extension.*
- E *is ideal if and only if* E *is admissible and it is contained in every preferred set of fuzzy arguments. The ideal extension is the maximal (w.r.t. set-inclusion) ideal set.*

5.4 The Division of Updated Fuzzy Argumentation Framework

The division of an FAF is based on the directionality principle of argumentation semantics. Notably, in this paper, if the attack relation has no influence in the FAF, i.e., the attack relation is always tolerable, then *we will not show this attack relation*. This can help us simplify the FAF. Given an $FAF = \langle \mathcal{A}, \rho \rangle$, for each pair arguments $A, B \in Ar$, if the attack relation from A to B is valid, i.e., A has influence on B, then we denote B is affected by A. Otherwise, B is independent of A. Based on this idea, the notion of reachability, as well as the notions of affected and unaffected between two arguments can be defined as follows:

Definition 14. *Let* $FAF = \langle \mathcal{A}, \rho \rangle$*, where* $Supp(\mathcal{A}) = Ar$*. The reachability of two arguments* $A, B \in Ar$ *w.r.t* ρ *is recursively defined as follows:*

- *If there exists* $(A, B) \in Supp(\rho)$*, then* B *is reachable from* A*;*
- *If* C *is reachable from* A*, and* B *is reachable from* C*, then* B *is reachable from* A*.*

Definition 15. *Let* $A, B \in Ar$*, and* ρ_{Ar} *be a set of fuzzy attacks within* Ar*. We say that under the semantics that satisfies the directionality principle,* B *is affected by* A*, iff* B *is reachable from* A *w.r.t.* ρ_{Ar}*. Otherwise,* B *is unaffected by* A *w.r.t* ρ_{Ar}*. In addition,* B *is affected by* $\mathcal{I}$*, iff* B *is reachable from an argument w.r.t.* $\mathcal{I}$*.*

Example 2. *Given an FAF with arguments A, B and C:*
$FAF = (\{(A, 0.6), (B, 0.7), (C, 0.8)\}, \{((A, B), 0.8), ((B, C), 0.6)\})$.
Here, from that $(A, 0.6)$ sufficiently attacks $(B, 0.7)$ and $(B, 0.7)$ sufficiently attacks $(C, 0.8)$, we have that B is reachable from A and C is reachable from B. Hence, C is reachable from A. From Definition 15, C is affected by A and B.

From the above definition, when an addition of FAF $\langle \mathcal{B}, \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2} \rangle$ is added to an FAF $\langle \mathcal{A}, \rho \rangle$, we can identify the subset of $\mathcal{A}$ which is affected by $\mathcal{B}$ or $\mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2}$. The initial FAF will be divided into three parts:

- a component of $\mathcal{A}$ that is affected by $(\mathcal{B}, \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2})$;
- a component of $\mathcal{A}$ that is unaffected by $(\mathcal{B}, \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2})$;
- a subset of the unaffected component that conditions the affected components.

Therefore, we are ready to define the notion of the division of an updated FAF. Formally, we can provide the division of an updated FAF w.r.t. an addition $(\mathcal{B}, \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2})$.

Definition 16. *Let $FAF = \langle \mathcal{A}, \rho \rangle$, and $Supp(\mathcal{A}) = Ar_1$. Suppose $(\mathcal{B}, \mathcal{I}_{Ar_1:Ar_2} \cup \mathcal{I}_{Ar_1})$ be an addition to the FAF. The updated FAF $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$ is divided into three parts: $\langle \mathcal{A}^{\oplus}_a, \rho^{\oplus}_a \rangle$, $\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u \rangle$, $(\mathcal{A}^{\oplus}_c, \rho^{\oplus}_c)$ where a, u and c stand for, respectively, affected, unaffected and conditioning.*
$\mathcal{A}^{\oplus}_a = \{(A, \mathcal{A}^{\oplus}(A)) \mid A \in Supp(\mathcal{B})$ *or A is affected by* $\mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2}$ *or A is affected by an argument* $C \in Supp(\mathcal{A}^{\oplus}_a)$ *w.r.t.* $\rho^{\oplus}\}$
$\mathcal{A}^{\oplus}_u = \mathcal{A}^{\oplus} - \mathcal{A}^{\oplus}_a$
$\mathcal{A}^{\oplus}_c = \{(A, \mathcal{A}^{\oplus}(A)) \in \mathcal{A}^{\oplus}_u \mid \exists B \in Supp(\mathcal{A}_a)$ *s.t.* $(A, B) \in Supp(\rho^{\oplus})$ *w.r.t.* $\rho^{\oplus}\}$
$\rho^{\oplus}_a = \rho^{\oplus} \cap \rho_{Supp(\mathcal{A}^{\oplus}_a)}$
$\rho^{\oplus}_u = \rho^{\oplus} \cap \rho_{Supp(\mathcal{A}^{\oplus}_u)}$
$\rho^{\oplus}_c = \rho^{\oplus} \cap \rho_{(Supp(\mathcal{A}^{\oplus}_c), Supp(\mathcal{A}^{\oplus}_a))}$

From this definition, for a given updated FAF $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$, $\mathcal{A}^{\oplus}_u$ coincides with the arguments that are unaffected by $(\mathcal{B}, \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2})$, $\mathcal{A}^{\oplus}_a$ coincides with the arguments that are affected by $(\mathcal{B}, \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2})$ as well as the fuzzy arguments in $\mathcal{B}$, $\mathcal{A}^{\oplus}_c$ coincides with the fuzzy arguments in $\mathcal{A}^{\oplus}_u$ that condition $\mathcal{A}^{\oplus}_a$.

After we have the division of the updated FAF, the next step is to construct two sub-frameworks of the updated FAF $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$: the unaffected FAF and the affected FAF under the condition. The unaffected FAF is $\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u \rangle$. And the conditioned FAF w.r.t. $\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u \rangle$ is constructed according to $\langle \mathcal{A}^{\oplus}_a, \rho^{\oplus}_a \rangle$ and $(\mathcal{A}^{\oplus}_c, \rho^{\oplus}_c)$ as follows:

$$CFAF = (\langle \mathcal{A}^{\oplus}_a, \rho^{\oplus}_a \rangle, (\mathcal{A}^{\oplus}_c, \rho^{\oplus}_c))$$

From the Definition 16, we have $\mathcal{A}^{\oplus}_c \cap \mathcal{A}^{\oplus}_a = \emptyset$, $\mathcal{A}^{\oplus}_c \subseteq \mathcal{A}^{\oplus}_u$ and $\rho^{\oplus}_c \subseteq \rho_{(\mathcal{A}^{\oplus}_c, \mathcal{A}^{\oplus}_a)}$. Namely, it satisfies the definition of condition.

Example 3. *Let $FAF = \langle \mathcal{A}, \rho \rangle$, in which $\mathcal{A} = \{(A, 0.8), (B, 0.7), (C, 0.7), (D, 0.6), (E, 0.8), (F, 0.6), (G, 0.7)\}$ and $\rho = \{((A, B), 0.8), ((A, C), 0.7), ((C, D), 0.6), ((C, F), 0.6), ((B, D), 0.9), ((D, E), 0.9), ((E, D), 0.7), ((F, G), 0.7)\}$.*

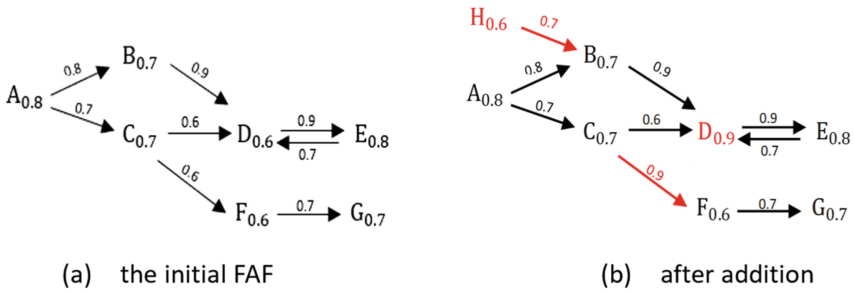

Fig. 2. An example of the division of a fuzzy argumentation framework (Example 3)

Let $(\mathcal{B}, \mathcal{I}_{Ar_1:Ar_2} \cup \mathcal{I}_{Ar_1})$ *be an addition FAF, in which* $\mathcal{B} = \{(D, 0.9), (H, 0.6)\}$, $Ar_1 = Supp(\mathcal{A})$, $Ar_2 = Supp(\mathcal{B})$, *and* $\mathcal{I}_{Ar_1:Ar_2} \cup \mathcal{I}_{Ar_1} = \{((C, F), 0.9), ((H, B), 0.7)\}$. *The updated FAF is* $\langle \mathcal{A} \cup \mathcal{B}, \rho \cup \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2}\rangle$*, in this example, the division of the updated FAF is showed as follows:*

- $\langle \mathcal{A}^{\oplus}_a, \rho^{\oplus}_a\rangle = \langle\{(B, 0.7), (D, 0.9), (E, 0.8), (F, 0.6), (G, 0.7), (H, 0.6)\}, \{((B, D), 0.9), ((D, E), 0.9), ((E, D), 0.7), ((F, G), 0.7), ((H, B), 0.7)\}\rangle$;
- $\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u\rangle = \langle\{(A, 0.8), (C, 0.7)\}, \{((A, C).0.7)\}\rangle$;
- $(\mathcal{A}^{\oplus}_c, \rho^{\oplus}_c) = (\{(A, 0.8), (C, 0.7)\}, \{((A, B), 0.8), ((C, D), 0.6), ((C, F), 0.9)$.

CFAF $= (\langle \mathcal{A}^{\oplus}_a, \rho^{\oplus}_a\rangle, (\mathcal{A}^{\oplus}_c, \rho^{\oplus}_c))$. *In this example, it is obvious that* $\langle \mathcal{A}^{\oplus}, \rho^{\oplus}\rangle$ *is equal to the combination of* $\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u\rangle$ *and* CFAF.

5.5 Computing the Semantics of an Updated Argumentation Framework Based on the Division

Under semantics $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{GR}\}$, based on the extensions of the two kinds of sub-frameworks, we will compute the extensions of $\langle \mathcal{A}^{\oplus}, \rho^{\oplus}\rangle$ by combining $\mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u\rangle)$ and $\mathcal{E}_{\mathcal{S}}(CFAF[E])$, in which $E \in \mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u\rangle)$.

Definition 17. *Let* $\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u\rangle$ *be the unaffected sub-framework of* $FAF = \langle \mathcal{A}, \rho\rangle$ *w.r.t an addition* $(\mathcal{B}, \mathcal{I}_{Ar_1} \cup \mathcal{I}_{Ar_1:Ar_2})$, $\mathcal{E}_{\mathcal{S}}\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u\rangle$ *be the set of extensions of* $\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u\rangle$*, and* $\text{CFAF}[E_1] = (\langle \mathcal{A}^{\oplus}_a, \rho^{\oplus}_a\rangle, (\mathcal{A}^{\oplus}_c[E_1], \rho^{\oplus}_c))$ *be an assigned conditioned sub-framework w.r.t.* $E_1 \in \mathcal{E}_{\mathcal{S}}\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u\rangle$. *The result of combining* $\mathcal{E}_{\mathcal{S}}\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u\rangle$ *and* $\mathcal{E}_{\mathcal{S}}(\text{CFAF}[E_1])$, $\forall E_1 \in \mathcal{E}_{\mathcal{S}}\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u\rangle$*, to form the set of combined extensions of* $(\langle \mathcal{A}^{\oplus}, \rho^{\oplus}\rangle)$*, denoted as* $CombExt_{\mathcal{S}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus}\rangle)$*, is defined as follows:*

$$CombExt_{\mathcal{S}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus}\rangle) = \{E_1 \cup E_2 \mid E_1 \in \mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}^{\oplus}_u, \rho^{\oplus}_u\rangle) \wedge E_2 \in \mathcal{E}_{\mathcal{S}}(\text{CFAF}[E_1])\}$$

Next, we will prove that under each semantic $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{GR}\}$, the extension of an updated FAF $\langle \mathcal{A}^{\oplus}, \rho^{\oplus}\rangle)$ coincides with the $CombExt_{\mathcal{S}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus}\rangle)$. Before the important theorem, we first figure out the relationship between a complete extension of an updated FAF and a complete extension of an assigned conditioned sub-framework of it. We have the following lemma:

Lemma 1. *For all $E \in \mathcal{E}_{\mathcal{CO}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle)$, it holds that $E \cap \mathcal{A}_a^{\oplus} \in \mathcal{E}_{\mathcal{CO}}(\mathrm{CFAF}[E_1])$, in which $E_1 = E \cap \mathcal{A}_u^{\oplus}$.*

Proof. Since complete semantics satisfies the directionality criterion, and $\mathcal{A}_u^{\oplus}$ is an unattacked set, according to Definition 12, it holds that $E_1 = E \cap \mathcal{A}_u^{\oplus} \in \mathcal{E}_{\mathcal{CO}}(\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle)$. According to the definition of assigned *CFAF*, it can be concluded that $E \cap \mathcal{A}_a^{\oplus} \subseteq \mathcal{A}_a^{\oplus}$ and $E \cap \mathcal{A}_a^{\oplus}$ is conflict-free. In order to prove that $E \cap \mathcal{A}_a^{\oplus}$ is a complete extension of *CFAF*$[E_1]$, we only need to verify the following two points:

- Every fuzzy argument in $E \cap \mathcal{A}_a^{\oplus}$ is weakening defended by $E \cap \mathcal{A}_a^{\oplus}$ under the condition $C(\mathcal{A}_u^{\oplus})[E_1]$, which is proved as follows:

  Since every fuzzy argument in $E \cap \mathcal{A}_a^{\oplus} \in E$ is weakening defended by E, it holds that $\forall (A, a) \in E \cap \mathcal{A}_a^{\oplus} \subseteq \mathcal{A}_a^{\oplus}$, if (B, b) sufficiently attacks (A, a), then there exists (C, c) in E s.t. $c * \rho_{CB} = a$. From the definition of $\mathcal{A}_a^{\oplus}$, (A, a) is only attacked by the fuzzy argument in $\mathcal{A}_a^{\oplus}$ and $\mathcal{A}_c^{\oplus}$. So, we have the following two cases:
  (i) If $(B, b) \in \mathcal{A}_a^{\oplus}$, then (B, b) is attacked by $\mathcal{A}_c^{\oplus}$ or $\mathcal{A}_a^{\oplus}$. It holds that $\exists (C, c)$ in $E \cap \mathcal{A}_c^{\oplus}$ s.t. $c * \rho_{CB} = a$ or in $E \cap \mathcal{A}_a^{\oplus}$ s.t. $c * \rho_{CB} = a$ (satisfying the first condition of weakening defense of fuzzy arguments in an assigned *CFAF*, in Definition 12).
  (ii) If $(B, b) \in \mathcal{A}_c^{\oplus}$, since the fuzzy argument in $\mathcal{A}_c^{\oplus}$ is only attacked by the fuzzy argument in $\mathcal{A}_u^{\oplus}$, we have that $\exists (C, c) \in E \cap \mathcal{A}_u^{\oplus} = E_1$ s.t. $c * \rho_{CB} = a$ (satisfying the second condition of weakening defense of fuzzy arguments in an assigned *CFAF*, in Definition 12).
- Every fuzzy argument which is weakening defended by $E \cap \mathcal{A}_a^{\oplus}$ under the condition $C(\mathcal{A}_u^{\oplus})[E_1]$ is in $E \cap \mathcal{A}_a^{\oplus}$, which is proved as follows:
  Since (A, a) in $\mathcal{A}_a^{\oplus}$ is attacked by $\mathcal{A}_c^{\oplus}$ or $\mathcal{A}_a^{\oplus}$, when (A, a) is weakening defended by $E \cap \mathcal{A}_a^{\oplus}$ under the condition $C(\mathcal{A}_u^{\oplus})[E_1]$, we have the following two cases:
  (i) If (B, b) in $\mathcal{A}_a^{\oplus}$ sufficiently attacks (A, a), then according to the first condition of weakening defense of fuzzy arguments in Definition 12, there exists $(C, c) \in E \cap \mathcal{A}_a^{\oplus} \subseteq E$ s.t. $c * \rho_{CB} = a$ or $(D, d) \in E_1 \cap \mathcal{A}_c^{\oplus} \subseteq E$ s.t. $d * \rho_{DB} = a$.
  (ii) If (B, b) in $\mathcal{A}_c^{\oplus}$ sufficiently attacks (A, a), then according to the second condition of weakening defense of fuzzy arguments in Definition 12, there exists $(C, c) \in E_1 \subseteq E$ s.t. $c * \rho_{CB} = a$.
  Consequently, for any (B, b) sufficiently attacks (A, a), there exists (C, c) in E s.t. $c * \rho_{CB} = a$. Therefore, (A, a) is weakening defended by E. According to the definition of complete extension, every fuzzy argument in $\mathcal{A}_a^{\oplus} \subseteq \mathcal{A}^{\oplus}$ that is weakening defended by E is in E, it holds that $(A, a) \in E$. Since $(A, a) \notin E_1$, it holds that $(A, a) \in E \cap \mathcal{A}_a^{\oplus}$.

Thus for all $E \in \mathcal{E}_{\mathcal{CO}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle)$, it holds that $E \cap \mathcal{A}_a^{\oplus} \in \mathcal{E}_{\mathcal{CO}}(CFAF[E_1])$, in which $E_1 = E \cap \mathcal{A}_u^{\oplus}$. □

Based on the Lemma 1, we first show that the combined extensions are semantics extensions of the updated FAF. The result is formulated in the following theorem.

Theorem 1. *Under each argumentation semantics* $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{GR}\}$, $\forall E \in CombExt_{\mathcal{S}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle)$*, it holds that* $E \in \mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle)$*, in which* $E = E_1 \cup E_2$*, an extension by combining* $E_1 \in \mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle)$ *and* $E_2 \in \mathcal{E}_{\mathcal{S}}(CFAF[E_1])$*.*

Proof. Under complete semantics, let $E = E_1 \cup E_2$, where $E_1 \in \mathcal{E}_{\mathcal{CO}}(\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle)$ and $E_2 \in \mathcal{E}_{\mathcal{CO}}(CFAF[E_1])$. In order to prove that E is a complete extension of $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$, we need proof that: (1) E is conflict-free; (2) every fuzzy argument in E is weakening defended by E; (3) every fuzzy argument which is weakening defended by E is in E.

(1) First of all, E_1 and E_2 include no conflict which is entailed by the hypothesis $E_1 \in \mathcal{E}_{\mathcal{CO}}(\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle)$ and $E_2 \in \mathcal{E}_{\mathcal{CO}}(CFAF[E_1])$. In addition, $\forall (A, a) \in E_1 \subseteq \mathcal{A}_a^{\oplus}$, $\forall (B, b) \in E_2 \subseteq A_u^{\oplus}$, it holds that (B, b) does not sufficiently attack (A, a), for the reason that $A_u^{\oplus}$ is unaffected, and it also holds that (A, a) does not sufficiently attack (B, b). Otherwise, (B, b) is sufficiently attacked by a conditioning fuzzy argument that is accepted w.r.t. E_1. According to the second condition of acceptability of arguments in an assigned *CFAF*, (B, b) is not acceptable w.r.t. E_2 under the condition $C(\mathcal{A}_u^{\oplus})[E_1]$, i.e., $(B, b) \notin E_2$, contradicting $(B, b) \in E_2$. Thus E is conflict-free.

(2) We need prove that for any $(A, a) \in E$, if (B, b) sufficiently attacks (A, a), then there exists $(C, c) \in E$ s.t. $c * \rho_{CB} = a$, namely there exist elements in E weakening defends (A, a).
For any $(A, a) \in E$, there are two possible cases: $(A, a) \in E_1$ or $(A, a) \in E_2$.

(i) If $(A, a) \in E_1$, then $(A, a) \in \mathcal{A}_u^{\oplus}$. Thus (A, a) is only attacked by the fuzzy arguments in $\mathcal{A}_u$. Form the hypothesis $E_1 \in \mathcal{E}_{\mathcal{CO}}(\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle)$, (A, a) is weakening defended by E_1 in $\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle$. Therefore E weakening defends (A, a).

(ii) If $(A, a) \in E_2$, then $(A, a) \in \mathcal{A}_a^{\oplus}$ and (A, a) is weakening defended by E_2 under the condition $C(\mathcal{A}_u^{\oplus})[E_1]$ in $CFAF[E_1]$. If (B, b) sufficiently attacks (A, a), then $(B, b) \in \mathcal{A}_a^{\oplus}$ or $(B, b) \in C[\mathcal{A}_u^{\oplus}]$. Since (A, a) is weakening defended by E_2 under the condition $C(\mathcal{A}_u^{\oplus})[E_1]$ in $CFAF[E_1]$, it holds that:

(a) if $(B, b) \in C(\mathcal{A}_u^{\oplus})$, then from Definition 12, $\exists (C, c) \in E_1$ s.t. $c * \rho_{CB} = a$. Namely there exist elements in E weakening defends (A, a).

(b) if $(B, b) \in \mathcal{A}_a^{\oplus}$, then from Definition 12, $\exists (C, c) \in E$ s.t. $c * \rho_{CB} = a$, or $\exists (D, d) \in C(\mathcal{A}_1)$, s.t. (D, d) is weakening defended by E_1 and $d * \rho_{DB} = a$. Since E_1 is a complete extension, $(D, d) \in E_1$. Thus there exist elements in E weakening defends (A, a).

From (i) and (ii), it can be concluded that E weakening defends all the fuzzy arguments in E.

(3) We assume that $\exists (A, a) \in \mathcal{A}^{\oplus}$ s.t. (A, a) is weakening defended by E, but $(A, a) \notin E$.

(i) If $(A,a) \in \mathcal{A}_u^{\oplus}$, then (A,a) is only attacked by the fuzzy arguments in $\mathcal{A}_u^{\oplus}$. Since (A,a) is weakening defended by E, we have that for any $(B,b) \in \mathcal{A}_u$ sufficiently attacks (A,a), there exists $(C,c) \in E$ s.t. $c * \rho_{CB} = a$. From that the fuzzy arguments in $\mathcal{A}_u^{\oplus}$ are only attacked by the fuzzy arguments in $\mathcal{A}_u^{\oplus}$, we have that $(C,c) \in \mathcal{A}_u^{\oplus} \cap E = E_1$. Thus (A,a) is weakening defended by E_1. According to that E_1 is a complete extension of $\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle$, it can be concluded that $(A,a) \in E_1$. But $(A,a) \notin E$. Thus it holds that $(A,a) \notin E_1$. Contradiction!

(ii) If $(A,a) \in \mathcal{A}_a^{\oplus}$, then (A,a) is only attacked by the fuzzy arguments in $\mathcal{A}_a^{\oplus}$ or $C(\mathcal{A}_u^{\oplus})$. Since (A,a) is weakening defended by E, it holds that:

(a) If (A,a) is sufficiently attacked by a fuzzy argument (B,b) in $\mathcal{A}_a^{\oplus}$, then there exists $(C,c) \in E$ s.t. $c * \rho_{CB} = a$. It is obvious that $(C,c) \in E_1$ or E_2. Thus, if (B,b) sufficiently attacks (A,a), then $\exists (C,c) \in E_2$ s.t. $c * \rho_{CB} = a$, or $\exists (D,d) \in C(\mathcal{A}_u^{\oplus})$, s.t. (D,d) is weakening defended by E_1 and $d * \rho_{DB} = a$ (satisfying the first condition of weakening defense of fuzzy arguments in Definition 12).

(b) If (A,a) is sufficiently attacked by a fuzzy argument (B,b) in $C(\mathcal{A}_u^{\oplus})$, then there exists $(C,c) \in \mathcal{A}_u \cap E = E_1$ s.t. $c * \rho_{CB} = a$. Thus for any $(B,b) \in C(\mathcal{A}_u^{\oplus})$, if (B,b) sufficiently attacks (A,a), then $\exists (C,c) \in E_1$ s.t. $c * \rho_{CB} = a$ (satisfying the second condition of weakening defense of fuzzy arguments in Definition 12).

Consequently, (A,a) is weakening defended by E_2 under the condition $C(\mathcal{A}_u^{\oplus})[E_1]$. Since $(A,a) \notin E$, it holds that $(A,a) \notin E_2$. Contradicting that E_2 is a complete extension of $CFAF[E_1]$.

According to (i) and (ii), we have that for any (A,a) which is weakening defended by E is contained in E. Therefore, every fuzzy argument which is weakening defended by E is in E.

– Under the preferred semantics, $E = E_1 \cup E_2$ where $E_1 \in \mathcal{E}_{\mathcal{PR}}(\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle)$ and $E_2 \in \mathcal{E}_{\mathcal{PR}}(CFAF[E_1])$: since a preferred extension is also a complete extension, we only need to prove that E is a maximal complete extension (with respect to set inclusion) of $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$. Assume that E is not a maximal complete extension. Then there exists a preferred extension S of $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$ which strictly contains E. We suppose $S_1 = S \cap \mathcal{A}_u^{\oplus}$ and $S_2 = S \cap \mathcal{A}_a^{\oplus}$. Then from that $\mathcal{A}_a^{\oplus} \cap \mathcal{A}_u^{\oplus} = \emptyset$, we have that $S_1 \cap S_2 = \emptyset$. According to the directionality principle and the preferred semantics satisfy the directionality principle, from that $\mathcal{A}_u^{\oplus}$ is an unattacked set of $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$, we have that S_1 is a preferred extension of $\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle$. Thus if $E_1 \subsetneq S_1$, then contradicting that $E_1 \in \mathcal{E}_{\mathcal{PR}}(\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle)$. Therefore, $E_1 = S_1$, it follows that $E_2 \subsetneq S_2$. Since a preferred extension is also a complete extension, according to Lemma 1, it holds that S_2 is a complete extension of $CFAF[E_1]$. Contradicting that E_2 is a preferred extension of $CFAF[E_1]$. Consequently, we conclude that E is a maximal complete extension (i.e., preferred extension). Hence $E \in \mathcal{E}_{\mathcal{PR}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle)$.

– Under grounded semantics, $E = E_1 \cup E_2$ where $E_1 \in \mathcal{E}_{\mathcal{GR}}(\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle)$ and $E_2 \in \mathcal{E}_{\mathcal{GR}}(CFAF[E_1])$: since the grounded extension is also a complete extension, we only need to prove that E is a minimal complete extension (with respect to set inclusion) of $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$. Assume that E is not a minimal complete extension. Then there exists a grounded extension S of $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$ which is strictly contained by E. We suppose $S_1 = S \cap \mathcal{A}_u^{\oplus}$ and $S_2 = S \cap \mathcal{A}_a^{\oplus}$. Then from that $\mathcal{A}_a^{\oplus} \cap \mathcal{A}_u^{\oplus} = \emptyset$, we have that $S_1 \cap S_2 = \emptyset$. According to the directionality principle and the grounded semantics satisfy the directionality principle, from that $\mathcal{A}_u^{\oplus}$ is an unattacked set of $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$, we have that S_1 is a grounded extension of $\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle$. Thus if $S_1 \subsetneq E_1$, then contradicting that $E_1 \in \mathcal{E}_{\mathcal{GR}}(\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle)$. Therefore, $E_1 = S_1$, it follows that $S_2 \subsetneq E_2$. Since a grounded extension is also a complete extension, according to Lemma 1, it holds that S_2 is a complete extension of $CFAF[E_1]$. Contradicting that E_2 is a grounded extension of $CFAF[E_1]$. As a result, we may conclude that E is a minimal complete extension (i.e., grounded extension). Hence $E \in \mathcal{E}_{\mathcal{GR}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle)$. □

According to Lemma 1, and Theorem 1, we immediately obtain Lemma 2.

Lemma 2. *Under each semantics $\mathcal{S} \in \{\mathcal{PR}, \mathcal{GR}\}$, $\forall E \in \mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle)$, it holds that $E \cap \mathcal{A}_a^{\oplus} \in \mathcal{E}_{\mathcal{S}}(\mathrm{CFAF}[E_1])$, in which $E_1 = E \cap \mathcal{A}_u^{\oplus}$.*

Proof. From Lemma 1, under complete semantics, $E \cap \mathcal{A}_a^{\oplus} \in \mathcal{E}_{\mathcal{CO}}(CFAF[E_1])$.

As far as preferred semantics are concerned, we need to prove that $E \cap \mathcal{A}_a^{\oplus}$ is a maximal complete extension. If $\exists E_2 \in \mathcal{E}_{\mathcal{PR}}(CFAF[E_1])$ and $E \cap \mathcal{A}_a^{\oplus} \subsetneq E_2$, then it follows that $E = (E \cap \mathcal{A}_u^{\oplus}) \cup (E \cap \mathcal{A}_a^{\oplus}) \subsetneq E_1 \cup E_2$. According to the directionality principle and $E_1 = E \cap \mathcal{A}_u^{\oplus}$, we have that E_1 is a preferred extension of $\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle$. From Theorem 1, we have that $E_1 \cup E_2$ is a preferred extension of $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$. This contradicts to the fact that E is a preferred extension of $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$. Hence, $E \cap \mathcal{A}_a^{\oplus} \in \mathcal{E}_{\mathcal{PR}}(CFAF[E_1])$.

As far as grounded semantics are concerned, we need to prove that $E \cap \mathcal{A}_a^{\oplus}$ is a minimal complete extension. If $\exists E_2 \in \mathcal{E}_{\mathcal{GR}}(CFAF[E_1])$ and $E_2 \subsetneq E \cap \mathcal{A}_a^{\oplus}$, then it follows that $E_1 \cup E_2 \subsetneq (E \cap \mathcal{A}_u^{\oplus}) \cup (E \cap \mathcal{A}_a^{\oplus}) = E$. According to the directionality principle and $E_1 = E \cap \mathcal{A}_u^{\oplus}$, we have that E_1 is a grounded extension of $\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle$. From Theorem 1, we have $E_1 \cup E_2$ is a grounded extension of $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$. This contradicts to the fact that E is a grounded extension of $\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle$. Hence, $E \cap \mathcal{A}_a^{\oplus} \in \mathcal{E}_{\mathcal{GR}}(CFAF[E_1])$. □

Based on Lemmas 1 and 2, we show that the semantics extensions are the combined extension of the updated FAF. The result is formulated in the following theorem.

Theorem 2. *Under each semantics $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{GR}\}$, $\forall E \in \mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle)$, it holds that $E \in CombExt_{\mathcal{S}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle)$.*

Proof. Under each semantics $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{GR}\}$, $\forall E \in \mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle)$ let $E_1 = \mathcal{A}_u^{\oplus} \cap E$, and $E_2 = \mathcal{A}_a^{\oplus} \cap E$. It holds that $E = E_1 \cup E_2$. According to Definition 10, Lemmas 1 and 2, it holds that $E_1 \in \mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle)$ and $E_2 \in \mathcal{E}_{\mathcal{S}}(CFAF[E_1])$. According to Definition 17, it holds that $E \in CombExt_{\mathcal{S}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle)$. □

We give an example to illustrate the process of computing the extensions of an updated FAF by the division method.

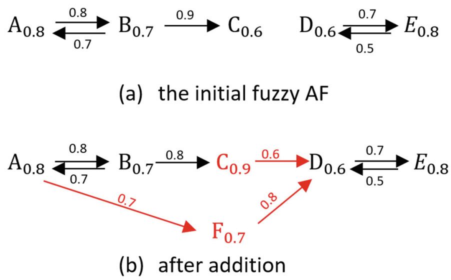

Fig. 3. The computation of semantics of an updated fuzzy argumentation framework (Example 4)

Example 4. *Let* $FAF = \langle \mathcal{A}, \rho \rangle$*, in which* $\mathcal{A} = \{(A, 0.8), (B, 0.7), (C, 0.6), (D, 0.6), (E, 0.8)\}$ *and* $\rho = \{((A, B), 0.8), ((B, A), 0.7), ((B, C), 0.9), ((D, E), 0.7), ((E, D), 0.5)\}$*. Let* $(\mathcal{B}, \mathcal{I}_{Ar_1:Ar_2} \cup \mathcal{I}_{Ar_1})$ *be an addition, in which* $\mathcal{B} = \{(C, 0.9), (F, 0.7)\}$*, and* $\mathcal{I}_{Ar_1:Ar_2} \cup \mathcal{I}_{Ar_1} = \{((A, F), 0.7), ((C, D), 0.6), ((F, D), 0.8)\}$*. Then, updated FAF is* $\langle \{(A, 0.8), (B, 0.7), (C, 0.9), (D, 0.6), (E, 0.8), (F, 0.7)\}, \{((A, B), 0.8), ((B, A), 0.7), ((B, C), 0.8), ((C, D), 0.6), ((D, E), 0.7), ((E, D), 0.5), ((A, F), 0.7), ((F, D), 0.8)\}\rangle$*, the division of the updated FAF is showed as follows:*

- $\langle \mathcal{A}_a^{\oplus}, \rho_a^{\oplus} \rangle = \langle \{(C, 0.9), (D, 0.6), (E, 0.8), (F, 0.7)\}, \{((F, D), 0.8), ((C, D), 0.6), ((D, E), 0.7), ((E, D), 0.5)\}\rangle$;
- $\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle = \langle \{(A, 0.8), (B, 0.7)\}, \{((A, B), 0.8), ((B, A), 0.7)\}\rangle$;
- $(\mathcal{A}_c^{\oplus}, \rho_c^{\oplus}) = (\{(A, 0.8), (B, 0.7)\}, \{((A, F), 0.7), ((B, C), 0.8)\})$.

We can obtain $\text{CFAF} = (\langle \mathcal{A}_a^{\oplus}, \rho_a^{\oplus} \rangle, (\mathcal{A}_c^{\oplus}, \rho_c^{\oplus}))$*. For simplicity, we only discuss the case under the preferred semantics. And we only consider the limit cases.*

Under preferred semantics, $\mathcal{E}_{\mathcal{PR}}(\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle) = \{E \cap \mathcal{A}_u^{\oplus} \mid E \in \mathcal{E}_{\mathcal{PR}}(\langle \mathcal{A}^{\oplus}, \rho^{\oplus} \rangle)\}$*. Two limit cases are* $E_1 = \{(A, 0.8), (B, 0.2)\}, E_2 = \{(A, 0.3), (B, 0.7)\}$*. Then we get two assigned* CFAFs: $\text{CFAF}[E_1], \text{CFAF}[E_2]$*. Next, we compute the preferred extensions of* $\text{CFAF}[E_1]$ *and* $\text{CFAF}[E_2]$ *according to Definitions 12 and 13. For simplicity, we only show a preferred extension* $\hat{E}_1$ *of* $\text{CFAF}[E_1]$ *and a preferred extension* $\hat{E}_2$ *of* $\text{CFAF}[E_2]$*, where* $\hat{E}_1 = \{(C, 0.8), (D, 0.4), (E, 0.6), (F, 0.3)\}$ *and* $\hat{E}_2 = \{(C, 0.3), (D, 0.3), (E, 0.7), (F, 0.7)\}$*. Finally, we combine the semantics extensions of* $\langle \mathcal{A}_u^{\oplus}, \rho_u^{\oplus} \rangle$ *and* CFAF*. From Theorem 1,* $E_1 \cup \hat{E}_1, E_2 \cup \hat{E}_2$ *are two preferred extensions of the updated FAF.*

5.6 The Conclusion About the Dynamics of FAF w.r.t. a Deletion of FAF

The dynamics of FAF have been explored when attached with an addition of FAF. In addition, from Definition 6, there exists the case of the deletion of

FAF. Indeed, we only need to explore the case of the removing of the arguments and attack relation. This is because the decrease of initial degree of arguments or attack relation can be regarded as we first remove the arguments or attack relation, and then we add the new belief degree of arguments or attack relation to the FAF.

Since the case of deletion of FAFs is similar to the addition of FAFs, we only list some definitions and theorems as follows and the proof procedure is omitted. And we only provide the case of the removing of arguments and attack relation.

Definition 18. *Let* $FAF = \langle \mathcal{A}, \rho \rangle$, *where* $\mathcal{A} : Ar_1 \to (0,1]$ *and* $\rho : Ar_1 \times Ar_1 \to (0,1]$ *are total functions. A deletion of FAF is represented as a tuple* $(\mathcal{B}, \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2})$, *in which,* $\mathcal{B} \subseteq \mathcal{A}$ *is a set of fuzzy arguments to be removed and* $Supp(\mathcal{B}) = Ar_2$, $\forall A \in Ar_2$, $\mathcal{B}(A) = \mathcal{A}(A)$, $\mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2}$ *is a set of fuzzy attacks to be removed and* $\forall (A,B) \in Supp(\mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2})$, $\mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2}(A,B) = \rho(A,B)$.

Definition 19. *Let* $FAF = \langle \mathcal{A}, \rho \rangle$, *in which* $\mathcal{A} : Ar_1 \to (0,1]$ *and* $\rho : Ar_1 \times Ar_1 \to (0,1]$ *are total functions. Let* $(\mathcal{B}, \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2})$ *be a deletion. The updated FAF w.r.t.* $(\mathcal{B}, \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2})$ *is represented as follows:*

$$\langle \mathcal{A}, \rho \rangle \ominus (\mathcal{B}, \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2}) = \langle \mathcal{A} - \mathcal{B}, \rho - \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2} \rangle$$

From the above definition, given an updated FAF $\langle \mathcal{A} - \mathcal{B}, \rho - \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2} \rangle$ with a deletion of FAF $(\mathcal{B}, \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2})$, we can identify the subset of $\mathcal{A}$ which is affected by $\mathcal{B}$ or $\mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2}$. Therefore, we are ready to define the concept of the division of an updated FAF. When a deletion $(\mathcal{B}, \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2})$ is deleted from an $FAF = \langle \mathcal{A}, \rho \rangle$, the updated FAF will be divided into three parts:

- a component of $\mathcal{A}$ that is affected by $(\mathcal{B}, \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2})$;
- a component of $\mathcal{A}$ that is unaffected by $(\mathcal{B}, \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2})$;
- a subset of the unaffected component that conditions the affected components.

Formally, we can provide the definition of the division of an FAF w.r.t. an addition $(\mathcal{B}, \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2})$.

Definition 20. *Let* $FAF = \langle \mathcal{A}, \rho \rangle$, *and* $Supp(\mathcal{A}) = Ar_1$. *Suppose* $(\mathcal{B}, \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2})$ *be a deletion to the* FAF. *The updated FAF* $\langle \mathcal{A}^\ominus, \rho^\ominus \rangle$ *is divided into three parts:* $\langle \mathcal{A}_a^\ominus, \rho_a^\ominus \rangle$, $\langle \mathcal{A}_u^\ominus, \rho_u^\ominus \rangle$, $(\mathcal{A}_c^\ominus, \rho_c^\ominus)$ *where a, u and c stand for, respectively, affected, unaffected and conditioning.*

$\mathcal{A}_a^\ominus = \{(A, \mathcal{A}^\ominus(A)) \mid A$ *is affected by* $\mathcal{B}$ *w.r.t.* $\mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2}$ *or* A *is affected by* $\mathcal{I}_{Ar_1 \setminus Ar_2}$ *or* A *is affected by an argument in* $Supp(\mathcal{A}_a^\ominus)$ *w.r.t.* $\rho^\ominus\}$

$\mathcal{A}_u^\ominus = \mathcal{A}^\ominus - \mathcal{A}_a^\ominus$

$\mathcal{A}_c^\ominus = \{(A, \mathcal{A}^\ominus(A)) \in \mathcal{A}_u^\ominus \mid \exists B \in Supp(\mathcal{A}_a)$ *s.t.* $(B, A) \in Supp(\rho^\ominus)$ *w.r.t.* $\rho^\ominus\}$

$\rho_a^\ominus = \rho^\ominus \cap \rho_{Supp(\mathcal{A}_a^\ominus)}$

$\rho_u^\ominus = \rho^\ominus \cap \rho_{Supp(\mathcal{A}_u^\ominus)}$

$\rho_c^\ominus = \rho^\ominus \cap \rho_{(Supp(\mathcal{A}_c^\ominus), Supp(\mathcal{A}_a^\ominus))}$

In this definition, for a given updated FAF $\langle \mathcal{A}^{\ominus}, \rho^{\ominus} \rangle$, $\mathcal{A}_u^{\ominus}$ coincides with the arguments that are unaffected by $(\mathcal{B}, \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2})$, $\mathcal{A}_a^{\ominus}$ coincides with the arguments that are affected by $(\mathcal{B}, \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2})$ $\mathcal{A}_c^{\ominus}$ coincides with the arguments in $\mathcal{A}_u^{\ominus}$ that attack $\mathcal{A}_a^{\ominus}$.

After we have the division of the updated FAF, the next step is to construct two sub-frameworks of the updated FAF $\langle \mathcal{A}^{\ominus}, \rho^{\ominus} \rangle$: the unaffected FAF and the affected FAF under the condition. The unaffected FAF is $\langle \mathcal{A}_u^{\ominus}, \rho_u^{\ominus} \rangle$. And the conditioned FAF w.r.t. $\langle \mathcal{A}_u^{\ominus}, \rho_a^{\ominus} \rangle$ is constructed as:

$$CFAF = (\langle \mathcal{A}_a^{\ominus}, \rho_a^{\ominus} \rangle, (\mathcal{A}_c^{\ominus}, \rho_c^{\ominus}))$$

From Definition 20, we have $\mathcal{A}_c^{\ominus} \cap \mathcal{A}_a^{\ominus} = \emptyset$, $\mathcal{A}_c^{\ominus} \subseteq \mathcal{A}_u^{\ominus}$ and $\rho_c^{\ominus} \subseteq \rho_{(\mathcal{A}_c^{\ominus}, \mathcal{A}_a^{\ominus})}$. Namely, it satisfies the definition of condition.

Based on the extensions of the two kinds of sub-frameworks, we will compute the extensions of $\langle \mathcal{A}^{\ominus}, \rho^{\ominus} \rangle$ by combining $\mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}_u^{\ominus}, \rho_u^{\ominus} \rangle)$ and $\mathcal{E}_{\mathcal{S}}(CFAF[E])$, in which $E \in \mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}_u^{\ominus}, \rho_u^{\ominus} \rangle)$.

Definition 21. *Let $\langle \mathcal{A}_u^{\ominus}, \rho_u^{\ominus} \rangle$ be the unaffected sub-framework of $FAF = \langle \mathcal{A}, \rho \rangle$ w.r.t. a deletion $(\mathcal{B}, \mathcal{I}_{Ar_1 \setminus Ar_2} \cup \mathcal{I}_{Ar_1 \setminus Ar_2 : Ar_2})$, $\mathcal{E}_{\mathcal{S}} \langle \mathcal{A}_u^{\ominus}, \rho_u^{\ominus} \rangle$ be the set of extensions of $\langle \mathcal{A}_u^{\ominus}, \rho_u^{\ominus} \rangle$, and $\mathrm{CFAF}[E_1] = (\langle \mathcal{A}_a^{\ominus}, \rho_a^{\ominus} \rangle, (\mathcal{A}_c^{\ominus}[E_1], \rho_c^{\ominus}))$ be an assigned conditioned FAF w.r.t. $E_1 \in \mathcal{E}_{\mathcal{S}} \langle \mathcal{A}_u^{\ominus}, \rho_u^{\ominus} \rangle$. The result of combining $\mathcal{E}_{\mathcal{S}} \langle \mathcal{A}_u^{\ominus}, \rho_u^{\ominus} \rangle$ and $\mathcal{E}_{\mathcal{S}}(\mathrm{CFAF}[E_1])$, $\forall E_1 \in \mathcal{E}_{\mathcal{S}} \langle \mathcal{A}_u^{\ominus}, \rho_u^{\ominus} \rangle$, to form the set of combined extensions of $\langle \mathcal{A}^{\ominus}, \rho^{\ominus} \rangle$, denoted as $CombExt_{\mathcal{S}}(\langle \mathcal{A}^{\ominus}, \rho^{\ominus} \rangle)$, is defined as follows:*

$$CombExt_{\mathcal{S}}(\langle \mathcal{A}^{\ominus}, \rho^{\ominus} \rangle) = \{E_1 \cup E_2 \mid E_1 \in E_{\mathcal{S}}(\langle \mathcal{A}_u^{\ominus}, \rho_u^{\ominus} \rangle) \wedge E_2 \in E_{\mathcal{S}}(\mathrm{CFAF}[E_1])\}$$

Next, we prove that under each semantic $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{GR}\}$, the extension of an updated framework $\langle \mathcal{A}^{\ominus}, \rho^{\ominus} \rangle)$ coincides with the $CombExt_{\mathcal{S}}(\langle A^{\ominus}, R^{\ominus} \rangle)$. We have the following important conclusion.

Lemma 3. *Under each semantics $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{GR}\}$, $\forall E \in \mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}^{\ominus}, \rho^{\ominus} \rangle)$, it holds that $E \cap \mathcal{A}_a^{\ominus} \in \mathcal{E}_{\mathcal{S}}(\mathrm{CFAF}[E_1])$, in which $E_1 = E \cap \mathcal{A}_u^{\ominus}$.*

Based on Lemmas 3, the coincidence of the semantics extensions and the combined extensions can be showed as follows:

Theorem 3. *Under each argumentation semantics $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{GR}\}$, $\forall E \in CombExt_{\mathcal{S}}(\langle \mathcal{A}^{\ominus}, \rho^{\ominus} \rangle)$, it holds that $E \in \mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}^{\ominus}, \rho^{\ominus} \rangle)$, in which $E = E_1 \cup E_2$, an extension by combining $E_1 \in \mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}_u^{\ominus}, \rho_u^{\ominus} \rangle)$ and $E_2 \in \mathcal{E}_{\mathcal{S}}(\mathrm{CFAF}[E_1])$.*

Theorem 4. *Under each semantics $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{GR}\}$, $\forall E \in \mathcal{E}_{\mathcal{S}}(\langle \mathcal{A}^{\ominus}, \rho^{\ominus} \rangle)$, it holds that $E \in CombExt_{\mathcal{S}}(\langle \mathcal{A}^{\ominus}, \rho^{\ominus} \rangle)$.*

6 Conclusion

In this paper, we explore the dynamics of FAFs. The changing of the argument and attack relation as well as the initial belief degree of the arguments and attack relation is an intrinsic property of FAFs with the changes of observations, basic knowledge, and information.

First, we list the whole changes in the dynamics of FAFs. The dynamics of FAFs include not only the changes of arguments and attack relation but also the changes of initial belief degree of arguments and attack relation. Furthermore, the arbitrary combination of these cases is also a dynamic FAF. Additionally, our main task is to compute the semantics of the dynamics FAFs. We focus on the complete, preferred and grounded semantics by extending Liao's division-based approach. First, we divide the updated FAF into three parts: affected FAF, unaffected FAF, conditioned FAF. Then we compute the semantics of the affected FAFs under the conditioned FAF. Due to the directionality principle, the semantics of the unaffected AF are directly obtained from the previous FAF. Thus, this algorithm conserves part of the semantics in the previous FAF.

In the future, we will continue exploring the residual semantics of the dynamics of FAFs, such as stable semantics, ideal semantics. We also want to prove that a variety of principles are satisfied in FAFs, such as reinstatement principle and SCC-recursiveness principle. Then we can provide an incremental computation in FAFs which can efficiently compute the semantics by the topology-related properties.

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